

MATH 141: Midterm 2

Name: _____

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * **Remember to simplify each expression.**
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

1. Suppose $f(x) = \sqrt{x}$.

(a) What does the expression $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ represent?

(b) Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

for the given function $f(x)$. You must use this limit definition to receive credit.

(c) Find the equation of the tangent line of $f(x)$ at the point $(1, 1)$.

2. Short answer questions:

- (a) If a function $f(x)$ is continuous at $x = a$, must it be differentiable at $x = a$ as well? If not, draw a graph of a function that is continuous but not differentiable at $x = a$.

- (b) True or false:

$$f(x) = \sin(x) + \frac{x}{x+1}$$

is continuous on \mathbb{R} .

- (c) Given $f(x) = x$, find an equation of the normal line at $(3, 3)$.

3. Answer the following:

(a) Given a function $f(x)$, if

$$\lim_{x \rightarrow a} f(x) = \frac{0}{0}$$

what global factor do you need to manifest in the numerator and denominator and why?

(b) Find

$$\lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$$

(c) Find

$$\lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

4. Find the following derivatives. You are allowed to use the Differentiation Rules.

(a) $f(x) = \pi^2$

(b) $f(x) = x^2 \sin x$

(c) $f(x) = \frac{\sin(x^2)}{2 - \cos x}$

(d) $g(x) = \sqrt{\tan x^3}$

5. Given the implicit equation

$$\sqrt{xy} = x + y$$

Find $\frac{dy}{dx}$.

6. The kinetic energy of an object is $K = \frac{1}{2}mv^2$. If the object is accelerating at a rate of 9.8 m/s and the mass is 30 kilograms, how fast is the kinetic energy increasing when the speed is 30 meters per second?